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THE FRIEDRICHS EXTENSION OF REGULAR SYMMETRIC DIFFERENTIAL OPERATORS

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Abstract. We increase the class of regular symmetric differential operators and find, explicitly, the boundary conditions which determine the Friedrichs extension of each one of these symmetric differential operators.

Mathematics subject classification (2020): Primary 34B24, 34L15; Secondary 34B08, 34L05.
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