

## BUZANO, KREĀN AND CAUCHY–SCHWARZ INEQUALITIES

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*Abstract.* The Cauchy-Schwarz, Buzano and KreĀn inequalities are three inequalities about inner product. The main goal of this article is to present refinements of Buzano and Cauchy-Schwarz inequalities, and to present a new proof of a refined version of a KreĀn-type inequality. Applications that include Buzano-type inequalities for certain operators, operator norm and numerical radius inequalities of Hilbert space operators will be presented.

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