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A NOTE ON SOME CLASSES OF G–MATRICES

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Abstract. Let $\mathbf{M}_n$ be the set of all $n \times n$ real matrices. A nonsingular matrix $A \in \mathbf{M}_n$ is called a G-matrix if there exist nonsingular diagonal matrices D_1 and D_2 such that $A^{-T} = D_1 A D_2$. For fixed nonsingular diagonal matrices D_1 and D_2, let $\mathbb{G}(D_1, D_2) = \{A \in \mathbf{M}_n : A^{-T} = D_1 A D_2\}$, which is called a G-class. In this note, a characterization of $\mathbb{G}(D_1, D_2)$ is obtained and some properties of these G-classes are exhibited, such as conditions for equality of two G-classes. It is shown that $\mathbb{G}(D_1, D_2)$ has two or four connected components in $\mathbf{M}_n$ and that $\mathbb{G}_n = \bigcup_{D_1, D_2} \mathbb{G}(D_1, D_2)$, the set of all $n \times n$ G-matrices, has two connected components in $\mathbf{M}_n$. Sign patterns of the G-classes are also examined.

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