AN INTERPOLATION PROPERTY OF REFLECTIONS
INVOLVING ORTHOGONAL PROJECTIONS

YUAN LI, YANAN WANG AND HONGKE DU

Abstract. Let $\mathscr{H}$ be a complex Hilbert space. We consider the interpolation problem: describe the pair (W,L) of subspaces of $\mathscr{H}$ such that there is a reflection J on $\mathscr{H}$ satisfying $J(W) \subseteq L$. We show that two subspaces W,L have this interpolation property if and only if $dim(W \cap L^\perp) \leqslant dim(L \cap W^\perp)$, which is equivalent to that there exists a conjugation C on $\mathscr{H}$ such that $C(W) \subseteq L$. Moreover, we study the least upper bound of these interpolating reflections.

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