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ON AN INEQUALITY CONJECTURED BY BESENYEI AND PETZ

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Abstract. In this paper we investigate the inequality $\mathrm{Tr}(T \otimes I_2)\rho_{12}(\log_q \rho_{12} - \log_q \rho_1 \otimes I_2 - I_1 \otimes \log_q \rho_2) \geqslant 0$, where ρ_{12} is a density matrix and $0 \leqslant T \in \mathbb{M}_m(\mathbb{C})$. This inequality was conjectured by Besenyei and Petz in 2013, where it was proved to hold for the density matrices in $\mathbb{M}_2(\mathbb{C}) \otimes \mathbb{M}_2(\mathbb{C})$. Here we prove this inequality for the density matrices in $\mathbb{M}_m(\mathbb{C}) \otimes \mathbb{M}_n(\mathbb{C})$ using some elementary matrix methods. We also obtain some new inequalities related to the operators (matrices) in this inequality.

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