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GENERALIZING THE ANDO–HIAI INEQUALITY FOR SECTORIAL MATRICES

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Abstract. In this paper, we extend a remarkable norm inequality of Ando and Hiai in 1994 about comparing the power of geometric mean and the geometric mean of powers of two positive semidefinite matrices to the case of sectorial matrices. To this end, we develop several new matrix inequalities that compare the real part of sectorial matrices.

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