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MATRIX SPLITTING AND IDEALS IN $\mathcal{B}(\mathcal{H})$

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Abstract. We investigate the relationship between ideal membership of an operator and its pieces relative to several canonical types of partitions of the entries of its matrix representation with respect to a given orthonormal basis. Our main theorems establish that if T lies in an ideal $\mathcal{I}$, then $\sum P_nTP_n$ (or more generally $\sum Q_nTP_n$) lies in the arithmetic mean closure of $\mathcal{I}$ whenever $\{P_n\}$ (and also $\{Q_n\}$) is a sequence of mutually orthogonal projections; and in any basis for which T is a block band matrix, in particular, when in Patnaik–Petrovic–Weiss universal block tridiagonal form, then all the sub/super/main-block diagonals of T are in $\mathcal{I}$. And in particular, the principal ideal generated by this T is the finite sum of the principal ideals generated by each sub/super/main-block diagonal.

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