PROPER NONNEGATIVE SPLITTINGS OVER
PROPER CONES OF RECTANGULAR MATRICES

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Abstract. In this paper, we further investigate the proper nonnegative splittings of rectangular matrices. The concept of proper nonnegative splittings over proper cones of rectangular matrices is proposed. Convergence results for the proper double nonnegative splitting over proper cones of a rectangular matrix are established, and comparison theorems for the spectral radii of matrices arising from proper nonnegative splittings over proper cones of the same rectangular matrix or different rectangular matrices are presented. The results obtained in this paper extend the results of proper nonnegative splittings over field to ones over proper cones of rectangular matrices. For ill-posed linear systems, the regularized iterative method based on splittings over proper cones is introduced, and the application of research results of proper nonnegative splittings over proper cones in the ill-posed linear system is given.

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