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STANDARD OPERATOR JORDAN RINGS ON BANACH SPACES, AND THEIR DERIVATIONS

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Abstract. Let X be a real or complex Banach space, let $\mathscr{L}(X)$ denote the algebra of all bounded linear operators on X, and let $\mathscr{F}(X)$ stand for the ideal of $\mathscr{L}(X)$ consisting of those operators in $\mathscr{L}(X)$ having finite-dimensional range. We introduce *standard operator Jordan algebras* on X as those Jordan subalgebras of $\mathscr{L}(X)$ which contain $\mathscr{F}(X)$.

As main results, we prove the following:

— If X is a real or complex Banach space, if $\mathscr{A}$ is a standard operator Jordan algebra on X, and if D is an $\mathscr{L}(X)$-valued linear (Jordan) derivation of $\mathscr{A}$, then there exists $B \in \mathscr{L}(X)$ such that $D(A) = [B,A]$ for every $A \in \mathscr{A}$.

— Every standard operator Jordan algebra $\mathscr{A}$ has minimum norm topology, i.e. the topology of any algebra norm on $\mathscr{A}$ is greater than or equal to that of the operator norm.

— Surjective algebra homomorphisms from complete normed Jordan algebras to standard operator Jordan algebras are continuous.

Actually, the first of the results just quoted is discussed when $\mathscr{A}$ is merely a *standard operator Jordan ring* on X (i.e, a Jordan subring of $\mathscr{L}(X)$ containing $\mathscr{F}(X)$) and D is assumed to be additive (as linearity of D could have not a meaning in this setting). In turn, the last of the results quoted above remains true if the completeness of the starting normed algebras is substantially weakened.

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