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A NORM INEQUALITY FOR SOME SPECIAL FUNCTIONS

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Abstract. Let A,B be invertible positive operators on a complex separable Hilbert space $\mathscr{H}$ and X be an operator on $\mathscr{H}$ associated with a norm ideal corresponding to a unitarily invariant norm $|||\cdot|||$. We shall prove that

$$|||\Gamma(A)X - X\Gamma(B)||| \leqslant c(m,M)|||AX - XB|||$$

for all unitarily invariant norms $|||\cdot|||$, where $c(m,M)$ is a function of $m = min\{||A||,||B||\}$ and $M = \max\{||A||,||B||\}$, and Γ denotes the Gamma function. Further if f is a Bernstein function, we shall prove that

$$|||f(A)X - Xf(B)||| \leqslant f'(m)|||AX - XB|||.$$

This inequality supplements and unify all the results proved by a number of authors for operator monotone functions.

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