# DILATIONS OF LINEAR MAPS ON VECTOR SPACES

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*Abstract.* Dilation of linear maps on vector spaces has been recently introduced by Bhat, De, and Rakshit. This notion is a variant of vector space dilation introduced by Han, Larson, Liu, and Liu. We derive vector space versions of Wold decomposition, Halmos dilation, N-dilation, inter-twining lifting theorem and a variant of Ando dilation. It is noted further that unlike a kind of uniqueness of Halmos dilation of strict contractions on Hilbert spaces, vector space version of Halmos dilation cannot be characterized.

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## REFERENCES

[1] T. ANDÔ, *On a pair of commutative contractions*, Acta Sci. Math. (Szeged), 24: 88–90, 1963.

[2] WILLIAM ARVESON, *Dilation theory yesterday and today*, In A glimpse at Hilbert space operators, vol. 207 of Oper. Theory Adv. Appl., pages 99–123, Birkhäuser Verlag, Basel, 2010.

[3] B. V. RAJARAMA BHAT, SANDIPAN DE, AND NARAYAN RAKSHIT, *A caricature of dilation theory*, Adv. Oper. Theory, 6 (4): Paper No. 63, 2021.

[4] B. V. RAJARAMA BHAT AND MITHUN MUKHERJEE, *Two states*, To appear in Houston Journal of Mathematics, arXiv:1710.00180v3 [math.FA] 23 April 2020.

[5] TIRTHANKAR BHATTACHARYYA, *Dilation of contractive tuples: a survey*, In Surveys in analysis and operator theory (Canberra, 2001), vol. 40 of Proc. Centre Math. Appl. Austral. Nat. Univ., pages 89–126, Austral. Nat. Univ., Canberra, 2002.

[6] PETER G. CASAZZA, DEGUANG HAN, AND DAVID R. LARSON, *Frames for Banach spaces*, In The functional and harmonic analysis of wavelets and frames (San Antonio, TX, 1999), vol. 247 of Contemp. Math., pages 149–182, Amer. Math. Soc., Providence, RI, 1999.

[7] WOJCIECH CZAJA, *Remarks on Naimark's duality*, Proc. Amer. Math. Soc., 136 (3): 867–871, 2008.

[8] E. EGERVÁRY, *On the contractive linear transformations of n-dimensional vector space*, Acta Sci. Math. (Szeged), 15: 178–182, 1954.

[9] PAUL R. HALMOS, *Normal dilations and extensions of operators*, Summa Brasil. Math., 2: 125–134, 1950.

[10] DEGUANG HAN AND DAVID R. LARSON, *Frames, bases and group representations*, Mem. Amer. Math. Soc., 147 (697): x+94, 2000.

[11] DEGUANG HAN, DAVID R. LARSON, BEI LIU, AND RUI LIU, *Dilations of frames, operator-valued measures and bounded linear maps*, In Operator methods in wavelets, tilings, and frames, vol. 626 of Contemp. Math., pages 33–53, Amer. Math. Soc., Providence, RI, 2014.

[12] DEGUANG HAN, DAVID R. LARSON, BEI LIU, AND RUI LIU, *Operator-valued measures, dilations, and the theory of frames*, Mem. Amer. Math. Soc., 229 (1075): viii+84, 2014.

[13] DEGUANG HAN, DAVID R. LARSON, BEI LIU, AND RUI LIU, *Structural properties of homomorphism dilation systems*, Chin. Ann. Math. Ser. B, 41 (4): 585–600, 2020.

[14] DEGUANG HAN, DAVID R. LARSON, AND RUI LIU, *Dilations of operator-valued measures with bounded p-variations and framings on Banach spaces*, J. Funct. Anal., 274 (5): 1466–1490, 2018.

[15] B. S. KASHIN AND T. YU. KULIKOVA, *A remark on the description of frames of general form*, Mat. Zametki, 72 (6): 941–945, 2002.

[16] DAVID R. LARSON AND FRANCISZEK HUGON SZAFRANIEC, *Framings and dilations*, Acta Sci. Math. (Szeged), 79 (3–4): 529–543, 2013.

[17] ELIAHU LEVY AND ORR MOSHE SHALIT, *Dilation theory in finite dimensions: the possible, the impossible and the unknown*, Rocky Mountain J. Math., 44 (1): 203–221, 2014.

[18] TZON-TZER LU AND SHENG-HUA SHIOU, *Inverses of $2 \times 2$ block matrices*, Comput. Math. Appl., 43 (1–2): 119–129, 2002.

[19] M. NEUMARK, *Spectral functions of a symmetric operator*, Bull. Acad. Sci. URSS. Sér. Math. [Izvestia Akad. Nauk SSSR], 4: 277–318, 1940.

[20] M. A. NEUMARK, *On a representation of additive operator set functions*, C. R. (Doklady) Acad. Sci. URSS (N.S.), 41: 359–361, 1943.

[21] J. J. SCHÄFFER, *On unitary dilations of contractions*, Proc. Amer. Math. Soc., 6: 322, 1955.

[22] ORR MOSHE SHALIT, *Dilation Theory: A Guided Tour*, In Operator theory, functional analysis and applications, vol. 282 of Oper. Theory Adv. Appl., pages 551–623, Birkhäuser/Springer, Cham, 2021.

[23] W. FORREST STINESPRING, *Positive functions on $C^*$-algebras*, Proc. Amer. Math. Soc., 6: 211–216, 1955.

[24] BÉLA SZ.-NAGY, *Sur les contractions de l'espace de Hilbert*, Acta Sci. Math. (Szeged), 15: 87–92, 1953.

[25] BÉLA SZ.-NAGY AND CIPRIAN FOIAŞ, *The "lifting theorem" for intertwining oerators and some new applications*, Indiana Univ. Math. J., 20 (10): 901–904, 1971.

[26] BELA SZ.-NAGY, CIPRIAN FOIAS, HARI BERCOVICI, AND LASZLO KERCHY, *Harmonic analysis of operators on Hilbert space*, Universitext, Springer, New York, second edition, 2010.