Operators and Matrices
Volume 16, Number 2 (2022), 545–558
doi:10.7153/oam-2022-16-41

BIDIAGONAL DECOMPOSITIONS AND TOTAL POSITIVITY OF SOME SPECIAL MATRICES

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Abstract. The matrix $S = [1 + x_i y_j]_{i,j=1}^n$, $0 < x_1 < \cdots < x_n$, $0 < y_1 < \cdots < y_n$, has gained importance lately due to its role in powers preserving total nonnegativity. We give an explicit decomposition of S in terms of elementary bidiagonal matrices, which is analogous to the *Neville decomposition*. We give a bidiagonal decomposition of $S^{\circ m} = [(1 + x_i y_j)^m]$ for positive integers $1 \leqslant m \leqslant n-1$. We also explore the total positivity of Hadamard powers of another important class of matrices called *mean matrices*.

Mathematics subject classification (2020): 15B05, 15A23, 15B48.
Keywords and phrases: Totally positive matrices, totally nonnegative matrices, Hadamard powers, infinitely divisible matrices, bidiagonal decomposition, mean matrices.

REFERENCES

[1] A. BARRERAS, J. M. PEÑA, *Bidiagonal decompositions, minors and applications*, Electron. J. Linear Algebra. 2012, 25: 60–71.
[2] R. BHATIA, *Infinitely divisible matrices*, Am. Math. Mon. 2006, 113: 221–235.
[3] R. BHATIA, S. FRIEDLAND, T. JAIN, *Inertia of Loewner matrices*, Indiana Univ. Math. 2016, 65: 1251–1261.
[4] R. BHATIA, H. KOSAKI, *Mean matrices and infinite divisibility*, Linear Algebra Appl. 2007, 424: 36–54.
[5] C. W. CRYER, *Some properties of totally positive matrices*, Linear Algebra Appl. 1976, 15: 1–25.
[6] S. M. FALLAT, *Bidiagonal factorizations of totally nonnegative matrices*, Am. Math. Mon. 2001, 108: 697–712.
[7] S. M. FALLAT, C. R. JOHNSON, *Hadamard powers and totally positive matrices*, Linear Algebra Appl. 2007, 423: 420–427.
[8] S. M. FALLAT, C. R. JOHNSON, *Totally Nonnegative Matrices*, New Jersey (NJ): Princeton University Press, 2011.
[9] S. M. FALLAT, C. R. JOHNSON, *Total positivity of sums, Hadamard products and Hadamard powers: Results and counterexamples*, Linear Algebra Appl. 2017, 520: 242–259.
[10] M. FIEDLER, T. L. MARKHAM, *Consecutive-column and-row Properties of matrices and the Loewner-Neville factorization*, Linear Algebra Appl. 1997 Nov 15, 266: 243–59.
[11] C. H. FITZGERALD, R. A. HORN, *On fractional Hadamard powers of positive definite matrices*, J. Math. Anal. Appl. 1977, 61: 633–642.
[12] M. GASCA, J. M. PEÑA, *On factorizations of totally positive matrices*, In Total positivity and its applications (pp. 109–130), Springer, Dordrecht, 1996.
[13] P. GROVER, V. S. PANWAR, A. S. REDDY, *Positivity of some special matrices*, Linear Algebra Appl. 2020, 596: 203–215.
[14] R. A. HORN, *The theory of infinitely divisible matrices and kernels*, Trans. Am. Math. Soc. 1969, 136: 269–286.
[15] R. HUANG, J. LIU, *Bidiagonal factorizations with some parameters equal to zero*, Linear Algebra Appl. 2011, 434 (3): 730–40.
[16] R. HUANG, *A test and bidiagonal factorization for certain sign regular matrices*, Linear Algebra Appl. 2013, 438 (3): 1240–51.

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[17] T. JAIN, *Hadamard powers of some positive matrices*, Linear Algebra Appl. 2017, 528: 147–158.
[18] T. JAIN, *Hadamard powers of rank two, doubly nonnegative matrices*, Adv. Oper. Theory. 2020, 5: 839–849.
[19] C. R. JOHNSON, D. D. OLESKY, P. VAN DEN DRIESSCHE, *Elementary bidiagonal factorizations*, Linear Algebra Appl. 1999, 292 (1–3): 233–244.
[20] C. R. JOHNSON, O. WALCH, *Critical exponents: old and new*, Electron. J. Linear Algebra. 2012, 25: 72–83.
[21] S. KARLIN, *Total positivity, absorption probabilities and applications*, Trans. Am. Math. Soc. 1964, 111: 33–107.
[22] S. KARLIN, *Total Positivity*, Vol. 1. Stanford: Stanford University Press, 1968.
[23] A. KHARE, *Multiply positive functions, critical exponent phenomena, and the Jain-Karlin-Schoenberg kernel*, arXiv preprint arXiv:2008.05121.2020.
[24] M. K. KWONG, *Some results on matrix monotone functions*, Linear Algebra Appl. 1989, 118: 129–153.
[25] H. ORUÇ, G. M. PHILLIPS, *Explicit factorization of the Vandermonde matrix*, Linear Algebra Appl. 2000, 315 (1–3): 113–123.
[26] V. S. PANWAR, A. S. REDDY, *Positivity of Hadamard powers of a few band matrices*, Electron. J. Linear Algebra 2022, 38: 85–90.
[27] A. PINKUS, *Totally Positive Matrices*, Cambridge (UK): Cambridge University Press, 2010.
[28] T. SANO, S. TACHIBANA, *On Loewner and Kwong matrices*, Sci. Math. Jpn. 2012, 75: 335–338.
[29] I. J. SCHOENBERG, *On Pólya frequency functions. I. The totally positive functions and their Laplace transforms*, J. d'Analyse Math. 1951, 1: 331–374.
[30] I. J. SCHOENBERG, *On the zeros of the generating functions of multiply positive sequences and functions*, Ann. Math. 1955, 62: 447–471.
[31] T. SIMON, *Total positivity of a Cauchy kernel*, J. Approx. Theory. 2014, 184: 238–58.