Operators
and
Matrices
Volume 16, Number 3 (2022), 697–708 doi:10.7153/oam-2022-16-51

ON THE SOLVABILITY OF GENERALIZED SYLVESTER OPERATOR EQUATIONS

HUA WANG, XIAOLIN SUN AND JUNJIE HUANG*

Abstract. In this paper, some necessary and sufficient solvability conditions are established for the generalized Sylvester operator equations $AXB-CXD=E$, $AXB-CYD=E$ and $AX+YB+CZ=E$ on Hilbert spaces, respectively. Moreover, we give a solvability condition for the $*$-Sylvester operator equation $AX-X^*B=C$, which holds for finite matrices due to Wimmer (1994).

Mathematics subject classification (2020): 47A62, 15A24.
Keywords and phrases: Operator equation, Sylvester equation, solvability, $*$-Sylvester equation.

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www.ele-math.com
oam@ele-math.com