Operators and Matrices
Volume 16, Number 3 (2022), 709–732
doi:10.7153/oam-2022-16-52

A PSEUDOSPECTRAL MAPPING THEOREM FOR OPERATOR PENCILS

KRISHNA KUMAR G. AND JUDY AUGUSTINE

Abstract. Let X be a complex Banach space and $BL(X)$ is the Banach algebra of all bounded linear operators on X. For $A,B \in BL(X)$, $n \in \mathbb{Z}_+$, and $\varepsilon > 0$, we define the (n,ε)-pseudospectrum of linear operator pencil (A,B) as

$$\Lambda_{n,\varepsilon}(A,B) = \sigma(A,B) \cup \left\{ \lambda \in \mathbb{C} : \left\| (\lambda B - A)^{-2^n} \right\|^{\frac{1}{2^n}} \geqslant \varepsilon^{-1} \right\}.$$

Here $\sigma(A,B)$ denotes the spectrum of the linear operator pencil (A,B). This article establishes certain properties of (n,ε)-pseudospectrum of operator pencils. We prove the Spectral Mapping Theorem for operator pencils. We also find an analogue of the Spectral Mapping Theorem for pseudospectrum and (n,ε)-pseudospectrum of operator pencils. Some examples are provided to illustrate the findings.

Mathematics subject classification (2020): Primary 47A10, 47A56, 15A22; Secondary 15A09, 65J10, 47A30.

Keywords and phrases: operator pencil, spectrum, pseudospectrum, (n,ε)-pseudospectrum, spectral mapping theorem.

REFERENCES

[1] A. AMMAR, A. JERIBI AND K. MAHFOUDHI, *The condition pseudospectrum of a operator pencil*, Asian-Eur. J. Math. **14** (2021), no. 4, Paper No. 2150057, 12 pp. MR4224960.

[2] S. J. BERNAU, *An unbounded spectral mapping theorem*, J. Austral. Math. Soc. **8** (1968), 119–127. MR0226434.

[3] E. B. DAVIES, *Linear operators and their spectra*, Cambridge Studies in Advanced Mathematics, 106, Cambridge University Press, Cambridge, 2007. MR2359869.

[4] K. DHARA AND S. H. KULKARNI, *The (n,ε)-pseudospectrum of an element of a Banach algebra*, J. Math. Anal. Appl. **464** (2018), no. 1, 939–954. MR3794123.

[5] H. A. DIJKSTRA, M. J. MOLEMAKER, A. VAN DER PLOEG, AND E. F. F. BOTTA, *An efficient code to compute non-parallel steady flows and their linear stability*, Computers and Fluids, **24** (1995), no. 4, 415–434.

[6] C. ENGSTRÖM AND M. RICHTER, *On the spectrum of an operator pencil with applications to wave propagation in periodic and frequency dependent materials*, SIAM J. Appl. Math. **70** (2009), no. 1, 231–247. MR2505087.

[7] A. C. HANSEN, *On the approximation of spectra of linear operators on Hilbert spaces*, J. Funct. Anal. **254** (2008), no. 8, 2092–2126. MR2402104.

[8] A. C. HANSEN, *Infinite-dimensional numerical linear algebra: theory and applications*, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **466** (2010), no. 2124, 3539–3559. MR2734774.

[9] A. C. HANSEN, *On the solvability complexity index, the n-pseudospectrum and approximations of spectra of operators*, J. Amer. Math. Soc. **24** (2011), no. 1, 81–124. MR2726600.

[10] A. C. HANSEN AND O. NEVANLINNA, *Complexity issues in computing spectra, pseudospectra and resolvents*, in *Études opératorielles*, 171–194, Banach Center Publ., 112, Polish Acad. Sci. Inst. Math., Warsaw. MR3754078.

[11] A. KHELLAF et al., *The pseudo-spectrum of operator pencils*, Asian-Eur. J. Math. **13** (2020), no. 5, 2050100, 16 pp. MR4134208.

[12] G. KRISHNA KUMAR AND S. H. KULKARNI, *An analogue of the spectral mapping theorem for condition spectrum*, in *Concrete operators, spectral theory, operators in harmonic analysis and approximation*, 299–316, Oper. Theory Adv. Appl., 236, Birkhäuser/Springer, Basel. MR3203068.
[13] S.-H. LUI, *A pseudospectral mapping theorem*, Math. Comp. **72** (2003), no. 244, 1841–1854. MR1986807.
[14] K. MAHFOUDHI, *Generalized determinant pseudo-spectrum of matrix pencils*, J. Anal. **29** (2021), no. 1, 163–176. MR4232811.
[15] W. RUDIN, *Functional analysis*, second edition, International Series in Pure and Applied Mathematics, McGraw-Hill, Inc., New York, 1991. MR1157815.
[16] M. SEIDEL, *On (N,ε)-pseudospectra of operators on Banach spaces*, J. Funct. Anal. **262** (2012), no. 11, 4916–4927. MR2913691.
[17] L. N. TREFETHEN AND M. EMBREE, *Spectra and pseudospectra*, Princeton University Press, Princeton, NJ, 2005. MR2155029.
[18] J. L. M. VAN DORSSELAER, *Pseudospectra for matrix pencils and stability of equilibria*, BIT **37** (1997), no. 4, 833–845. MR1485868.