FROBENIUS—RIEFFEL NORMS ON
FINITE–DIMENSIONAL C*–ALGEBRAS

KONRAD AGUILAR, STEPHAN RAMON GARCIA AND ELENA KIM

Abstract. In 2014, Rieffel introduced norms on certain unital C*-algebras built from conditional expectations onto unital C*-subalgebras. We begin by showing that these norms generalize the Frobenius norm, and we provide explicit formulas for certain conditional expectations onto unital C*-subalgebras of finite-dimensional C*-algebras. This allows us compare these norms to the operator norm by finding explicit equivalence constants. In particular, we find equivalence constants for the standard finite-dimensional C*-subalgebras of the Effros–Shen algebras that vary continuously with respect to their given irrational parameters.

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