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STABILITY BOUNDS FOR RECONSTRUCTION FROM SAMPLING ERASURES

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Abstract. The Shannon-Whittaker Sampling Theorem states that a frequency bounded signal can be completely determined by its sampled values at a countable number of points. Thus, the theorem allows us to convert analog signals to digital signals by sampling (or evaluating) the signal at these points. In prior work, it was shown that if a signal is oversampled, and if some of the sampled values are lost when transmitting the signal, then it is still possible to reconstruct the signal. However, in certain situations, the reconstruction algorithm is very unstable. In this paper, we provide stability bounds on the reconstruction algorithm and determine when it is not feasible to perform the reconstruction.

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