

## ON TOEPLITZ-PLUS-HANKEL MATRICES AND TOEPLITZ-PLUS-HANKEL-BEZOUTIANS

TORSTEN EHRHARDT AND KARLA ROST

*Abstract.* The main aim of the present paper is to establish invertibility criteria for Toeplitz-plus-Hankel-Bezoutians based on the analysis of the structure of these matrices. In particular, their inverses have an explicit representation as a sum of a Toeplitz and a Hankel matrix whose symbols are the solution of certain linear systems taking the form of generalized resultant equations. These results generalize previous inversion formulas and criteria for the special cases of centrosymmetric or centroskewsymmetric Toeplitz-plus-Hankel-Bezoutians.

The inversion of Toeplitz-plus-Hankel-Bezoutians considered here is based on the converse problem, the inversion of Toeplitz-plus-Hankel matrices. Consequently, several modifications of known inversion formulas and new results for Toeplitz-plus-Hankel matrices are developed, which allow a deeper insight into the structure of these matrices, too.

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