Operators and Matrices
Volume 16, Number 3 (2022), 909–923
doi:10.7153/oam-2022-16-61

ON SOME ALGEBRAIC PROPERTIES OF BLOCK TOEPLITZ MATRICES WITH COMMUTING ENTRIES

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Abstract. Toeplitz matrices are ubiquitous and play important roles across many areas of mathematics. In this paper, we present some algebraic results concerning block Toeplitz matrices with block entries belonging to a commutative algebra $\mathscr{A}$. The characterization of normal block Toeplitz matrices with entries from a commutative algebra $\mathscr{A}$ of normal matrices is also obtained.

Mathematics subject classification (2020): 15A27, 15A30, 15B05.
Keywords and phrases: Block Toeplitz matrices, displacement matrix, normal matrices.

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Paper OaM-16-61

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