NOTES ON MAJORIZATIONS FOR SINGULAR VALUES

JIANGUO ZHAO

Abstract. In this note, we mainly investigate the majorizations on the products and sums of matrices. Firstly, we present the following result: Let $A_i, B_i$ and $X_i \in M_n(C)$ ($i = 1, 2, \cdots, m$) with $X_i$ ($i = 1, 2, \cdots, m$) are invertible matrices, and let $h$ be a nonnegative increasing continuous function on $[0, +\infty)$ with $h(0) = 0$. If $f, g$ are nonnegative continuous functions on $[0, +\infty)$ with $f(t)g(t) = t$ for $t \in [0, +\infty)$, then

$$\alpha \circ s\left(\sum_{i=1}^{m} A_i X_i |X_i|^{-1} h(|X_i|) B_i\right) \prec_w \alpha \circ \left\{ \frac{1}{p} s\left(\sum_{i=1}^{m} A_i f^2(h(|X_i|)) A_i^* \right)^{\alpha_i} + \frac{1}{q} s\left(\sum_{i=1}^{m} B_i g^2(h(|X_i|)) B_i^* \right)^{\alpha_i} \right\},$$

where $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$, $p$, $q$, $r$ and $\alpha_i$ ($i = 1, 2, \cdots, n$) are positive real numbers with $\frac{1}{p} + \frac{1}{q} = 1$. Then, some other weak majorizations are given. These obtained inequalities directly generalize the results obtained by Huang [H. Huang, On majorizations and singular values, Linear Multilinear Algebra, (2020), https://doi.org/10.1080/03081087.2020.1836117].


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REFERENCES