NOTES ON MAJORIZATIONS FOR SINGULAR VALUES

JIANGUO ZHAO

Abstract. In this note, we mainly investigate the majorizations on the products and sums of matrices. Firstly, we present the following result: Let A_i, B_i and $X_i \in M_n(\mathscr{C})$ $(i = 1, 2, \cdots, m)$ with X_i $(i = 1, 2, \cdots, m)$ are invertible matrices, and let h be a nonnegative increasing continuous function on $[0, +\infty)$ with $h(0) = 0$. If f, g are nonnegative continuous functions on $[0, +\infty)$ with $f(t)g(t) = t$ for $t \in [0, +\infty)$, then

$$\alpha \circ s \left(\left| \sum_{i=1}^m A_i X_i |X_i|^{-1} h(|X_i|) B_i \right|^r \right)$$

$$\prec_w \alpha \circ \left\{ \frac{1}{p} s \left(\left(\sum_{i=1}^m A_i f^2(h(|X_i^*|)) A_i^* \right)^{\frac{pr}{2}} \right) + \frac{1}{q} s \left(\left(\sum_{i=1}^m B_i^* g^2(h(|X_i|)) B_i \right)^{\frac{qr}{2}} \right) \right\},$$

where $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$, p, q, r and α_i $(i = 1, 2, \cdots, n)$ are positive real numbers with $\frac{1}{p} + \frac{1}{q} = 1$. Then, some other weak majorizations are given. These obtained inequalities directly generalize the results obtained by Huang [H. Huang, On majorizations and singular values, Linear Multilinear Algebra, (2020), https://doi.org/10.1080/03081087.2020.1836117].

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