Operators and Matrices
Volume 16, Number 4 (2022), 1005–1026
doi:10.7153/oam-2022-16-67

ON THE A_α SPECTRAL RADIUS OF STRONGLY CONNECTED DIGRAPHS

WEIGE XI AND LIGONG WANG*

Abstract. Let $A(G)$ and $D(G)$ be the adjacency matrix and the diagonal matrix with outdegrees of vertices of a digraph G, respectively. In 2017, Nikiforov proposed to study the convex combinations of the adjacency matrix and diagonal matrix of the degrees of undirected graphs. In 2019, Liu et al. extended the definition to digraphs. For any real $\alpha \in [0,1]$, the matrix $A_\alpha(G)$ of a digraph G is defined as $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G)$. The largest modulus of the eigenvalues of $A_\alpha(G)$ is called the A_α spectral radius of G, denoted by $\lambda_\alpha(G)$. This paper proves some extremal results about the A_α spectral radius $\lambda_\alpha(G)$ that generalize previous results about $\lambda_0(G)$ and $\lambda_{\frac{1}{2}}(G)$. We mainly characterize the extremal digraph with the maximum (or minimum) A_α spectral radius among all $\widetilde{\infty}$-digraphs and $\widetilde{\theta}$-digraphs on n vertices. Furthermore, we determine the digraphs with the second and the third minimum A_α spectral radius among all strongly connected bicyclic digraphs. For $0 \leqslant \alpha \leqslant \frac{1}{2}$, we also determine the digraphs with the second, the third and the fourth minimum A_α spectral radius among all strongly connected digraphs on n vertices. Finally, we characterize the digraph with the minimum A_α spectral radius among all strongly connected bipartite digraphs which contain a complete bipartite subdigraph.

Mathematics subject classification (2020): 05C50,15A18.
Keywords and phrases: Strongly connected digraph, signless Laplacian matrix, adjacency matrix, A_α spectral radius.

REFERENCES

[1] A. BERMAN, R. J. PLEMMONS, *Nonnegative Matrices in the Mathematical Sciences*, New York: Academic Press, 1979.
[2] S. T. CHEN, S. L. CHEN, W. Q. LIU, *The minimum spectral radius of strongly connected bipartite digraphs with complete bipartite subdigraph*, Quantitative Logic and Soft Computing 2016, Springer International Publishing 2017, 659–669.
[3] H. A. GANIE, M. BAGHIPUR, *On the generalized adjacency spectral radius of digraphs*, Linear Multilinear Algebra, 70 (2022), 3497–3510.
[4] G. Q. GUO, J. LIU, *Some results on the spectral radius of generalized ∞ and θ-digraphs*, Linear Algebra Appl., 437 (2012), 2200–2208.
[5] R. A. HORN, C. R. JOHNSON, *Matrix Analysis*, Cambridge University Press, New York, 1985.
[6] W. X. HONG, L. H. YOU, *Spectral radius and signless Laplacian spectral radius of strongly connected digraphs*, Linear Algebra Appl., 457 (2014), 93–113.
[7] J. LI, B. ZHOU, *On the spectral radius of strongly connected digraphs*, Bull Iranian Math. Soc., 41 (2015), 381–387.
[8] H. Q. LIN, S. W. DRURY, *The maximum perron roots of digraphs with some given parameters*, Discrete Math., 313 (2013), 2607–2613.
[9] X. H. LI, L. G. WANG S. Y ZHANG, *The signless Laplacian spectral radius of some strongly connected digraphs*, Indian J. Pure Appl. Math., 49 (2018), 113–127.
[10] H. Q. LIN, J. L. SHU, *Spectral radius of digraphs with given dichromatic number*, Linear Algebra Appl., 434 (2011), 2462–2467.
[11] H. Q. LIN, J. L. SHU, *A note on the spectral characterization of strongly connected bicyclic digraphs*, Linear Algebra Appl., 436 (2012), 2524–2530.

Paper OaM-16-67

[12] H. Q. LIN, J. L. SHU, Y. R. WU, G. L. YU, *Spectral radius of strongly connected digraphs*, Discrete Math., 312 (2012), 3663–3669.
[13] H. Q. LIN, X. G. LIU, J. XUE, *Graphs determined by their A_α-spectra*, Discrete Math., 342 (2019), 441–450.
[14] J. P. LIU, X. Z. W, J. S. CHEN, B. L. LIU, *The A_α spectral radius characterization of some digraphs*, Linear Algebra Appl., 563 (2019), 63–74.
[15] X. G. LIU, S. Y. LIU, *On the A_α-characteristic polynomial of a graph*, Linear Algebra Appl., 546 (2018), 274–288.
[16] V. NIKIFOROV, *Merging the A- and Q-spectral theories*, Applicable Analysis and Discrete Math., 11 (2017), 81–107.
[17] V. NIKIFOROV, O. ROJO, *On the α-index of graphs with pendent paths*, Linear Algebra Appl., 550 (2018), 87–104.
[18] V. NIKIFOROV, O. ROJO, *A note on the positive semidefiniteness of $A_\alpha(G)$*, Linear Algebra Appl., 519 (2017), 156–163.
[19] W. G. XI, L. G. WANG, *Sharp upper bounds on the signless Laplacian spectral radius of strongly connected digraphs*, Discuss. Math. Graph Theory, 36 (2016), 977–988.
[20] W. G. XI, L. G. WANG, *The signless Laplacian and distance signless Laplacian spectral radius of digraphs with some given parameters*, Discrete Appl. Math., 227 (2017), 136–141.
[21] W. G. XI, W. SO, L. G. WANG, *On the A_α spectral radius of digraphs with given parameters*, Linear Multilinear Algebra, 70 (2022), 2248–2263.
[22] J. XUE, H. Q. LIN, S. T. LIU, J. L. SHU, *On the A_α-spectral radius of a graph*, Linear Algebra Appl., 550 (2018), 105–120.