

WEAK-STAR DENTABILITY, QUASI-WEAK-STAR NEAR DENTABILITY AND CONTINUITY OF METRIC PROJECTOR IN BANACH SPACES

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Abstract. The relations between the w^* dentability and Chebyshev set or the continuity of metric projection operator are given. Let X^* be the conjugate space of Banach space X , the conditions of a point (x^*, y^*) on the unit sphere of product space $X^* \times X^*$ to be w^* denting point of closed unit ball of product space $X^* \times X^*$ are given. Also, a notion of quasi- w^* near dentability in conjugate space X^* is introduced and the relations between the quasi- w^* nearly denting point of closed unit ball of X^* and a certain slice of closed unit ball of X^* are given.

Mathematics subject classification (2020): 46B09, 46B20.

Keywords and phrases: Banach space, slice, w^* denting point, quasi- w^* nearly denting point, Chebyshev set, metric projection operator.

REFERENCES

- [1] M. ABBAS AND S. Z. NÉMETH, *Solving nonlinear complementarity problems isotonicity of the metric projection*, Journal of Mathematical Analysis and Applications **386** (2) (2012) 882–893.
- [2] D. Q. CHEN, *A sufficient condition for the uniqueness of support functions*, Sci. Sin. Math. (in Chinese), **25** (3) (1982) 302–305.
- [3] S. T. CHEN, H. HUDZIK, W. KOWALEWSKI, Y. W. WANG, AND M. WISLA, *Approximative compactness and continuity of metric projector in Banach spaces and applications*, Science in China, Series A: Mathematics **51** (2) (2008) 293–303.
- [4] K. FAN AND I. GLICKSBURG, *Some geometric properties of the spheres in a normed linear space*, Duke. Math. J. **25** (1958) 553–568.
- [5] X. N. FANG AND J. H. WANG, *Convexity and the continuity of metric projections*, Mathematica Applicata (PRC) (in Chinese), **14** (1) (2001) 47–51.
- [6] X. N. FANG AND J. H. WANG, *Slice and convexity, smoothness of Banach spaces*, J. Math. (PRC) (in Chinese), **19** (3) (1999) 293–298.
- [7] N. W. JEFIMOW AND S. B. STECHKIN, *Approximative compactness and Chebyshev sets*, Soviet. Mathematics. **2** (1961) 1226–1228.
- [8] V. ISTRATESCU, *Strict convexity and complex strict convexity: theory and applications*, Dekker. New York, 1984.
- [9] D. Z. KONG, L. S. LIU, W. H. WU, *Isotonicity of the metric projection with applications to variational inequalities and fixed point theory in Banach spaces*, Journal of fixed point theory and applications **19** (2017) 1889–1903.
- [10] J. L. LI, *The generalized projection operator on reflexive Banach spaces and its applications*, Journal of Mathematical Analysis and Applications **306** (1) (2005) 55–71.
- [11] J. L. LI, C. J. ZHANG, X. H. MA, *On the metric projection operator and its applications to solving variational inequalities in Banach spaces*, Numerical Functioal Analysis and Optimization **29** (3–4) (2008) 410–418.
- [12] J. LINDENSTRAUSS, *On operators which attain their norm*, Isreal J. Math. **(3)** (1963) 139–148.
- [13] P. D. LIU, *Martingle and geometric of Banach space*, (in Chinese), Science Press, Shanghai, 2007.
- [14] H. B. MAYNARD, *A geometrical characterization of Banach spaces having the Radon-Nikodym property*, Trans. Amer. Math. Soc. **185** (1973) 493–500.
- [15] R. E. MEGGINSON, *An introduction to Banach spaces*, Springer-Verlag, New York, Inc., 1998.

- [16] V. MONTESIONS, *On drop property*, Studia Math. **85** (1987) 25–35.
- [17] C. X. NAN, *On the weakly exposed points*, Northeastern Math. J. **8** (4) (1990) 449–454.
- [18] E. V. OSHMAN, *Characterization of subspaces with continuous metric projection into a normed linear space*, Soviet Mathematics **13** (1972) 1521–1524.
- [19] B. B. PANDA O. P. KAPOOR, *A generalization of local uniform convexity of the norm*, J. Math. Anal. Appl. **52** (3) (1975) 300–308.
- [20] R. R. PHELPS, *Convex functions, monotone operators and differentiability*, Lecture Notes, in Math, Springer-Verlag, New York, 1989.
- [21] H. ROBERT AND L. WU, *Continuities of metric projection and geometric consequences*, Journal of Approximation Theory **90** (3) (1997) 319–339.
- [22] M. A. RIEFFEL, *Dentable subsets of Banach spaces, with applications to a Radon-Nikodym theorem*, Funct. Anal. Proc. (Conf. Irvine, Calif., 1966), Acad. Press, London, Thompson, Washington, D.C., 71–77.
- [23] S. Q. SHANG, Y. A. CUI, *Approximative compactness and continuity of the set-valued metric generalized inverse in Banach spaces*, J. Math. Anal. Appl. **422** (2015) 1363–1375.
- [24] S. Q. SHANG AND Y. A. CUI, *Approximative Compactness and Radon-Nikodym Property in w^* nearly dentable Banach spaces and applications*, Journal of Function Spaces (2015), Article no. 277355713.
- [25] S. Q. SHANG, Y. A. CUI, Y. Q. FU, *Dentable point and strongly smoothness and approximation compactness in Banach spaces*, Acta Mathematica Sinica (in Chinese) **53** (6) (2010) 1217–1224.
- [26] S. Q. SHANG, Y. A. CUI, Y. Q. FU, *Nearly dentability and approximative compactness and continuity of metric projector in Banach spaces*, Sci Sin Math (in Chinese) **41** (9) (2011) 815–825.
- [27] S. Q. SHANG AND J. X. ZHANG, *Metric projection operator and continuity of the set-valued metric generalized inverse in Banach spaces*, Journal of Function Spaces (2017), Article no. 7151430.
- [28] J. H. WANG, *Some results on the continuity of metric projections*, Mathematica Applicata (PRC) **8** (1) (1995) 80–84.
- [29] C. X. WU AND Y. J. LI, *Strong convexity in Banach spaces*, J. Math (PRC) **13** (1) (1993) 105–108.
- [30] C. X. WU AND Y. J. LI, *Dentability and extreme points*, Northeastern Math J. (PRC) **9** (3) (1993) 305–307.
- [31] S. Y. XU, C. LI, W. S. YANG, *Non-linear approximation in Banach space*, (in Chinese), Beijing: Science Press, 1998.
- [32] Z. B. XU AND G. F. ROACH, *On the uniform continuity of metric projections in Banach spaces*, Approximation Theory and Its Appl. **8** (3) (1992) 11–20.
- [33] X. T. YU, *Geometric theory of Banach space*, (in Chinese), Shanghai: East China, Normal University Press, 1984.