# SPECTRA OF GRAPHENES WITH VARIANT EDGES

## Hiroaki Niikuni

*Abstract.* In this paper, we consider a periodic quantum graph corresponding to graphene with a variant of the zigzag shape of boundaries. The aim of this paper is to compare the spectra of our graphs with the spectra of quantum graphs with the standard zigzag boundaries. For this purpose, we utilize a Shnol type theorem and the Cramer's rule to construct two spectral discriminants $D_s(\mu, \lambda)$ and $D_c(\mu, \lambda)$, where $\mu = S^1 := [-\pi, \pi)$ is a quasi-momentum of a corresponding fiber operator and $\lambda \in \mathbb{R}$ is a spectral parameter. As a result, we derive pictures of a part of the dispersion relation for our quantum graph.

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