

EXPLICIT SOLUTIONS OF MATRIX AND DYNAMICAL SCHRÖDINGER EQUATIONS AND OF KDV EQUATION IN TERMS OF SQUARE ROOTS OF THE GENERALISED MATRIX EIGENVALUES

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Abstract. In this paper, we consider matrix Schrödinger equation, dynamical Schrödinger equation and matrix KdV. We construct their explicit solutions using our GBDT version of Bäcklund–Darboux transformation and square roots of the generalised matrix eigenvalues. A separate section is dedicated to several examples including the case of strongly singular potentials.

Mathematics subject classification (2020): 34A05, 34L40, 35Q53, 37C79, 15A16, 15A24.

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