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A GENERALIZATION OF KLEINECKE–SHIROKOV THEOREM FOR MATRICES

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Abstract. For given square matrices A and B we denote by $Y = AB - BA$ and by $Z = AY - YA$. It is well known that if A and Y commute, i.e., if $Z = 0$, then Y is a nilpotent matrix. In this note we show that the same is true if $YZ = ZY$. We also generalize this result by using commutators of higher order.

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REFERENCES

[1] J. BRAČIČ AND B. KUZMA, *Localizations of the Kleinecke-Shirokov Theorem*, Oper. Matrices **1** (2007), 385–389.
[2] R. A. HORN AND C. R. JOHNSON, *Topics in Matrix Analysis*, Cambridge University Press, Cambridge 1991.
[3] N. JACOBSON, *Rational methods in the theory of Lie algebras*, Ann. of Math. **36** (1935), 875–881.
[4] D. C. KLEINECKE, *On operator commutators*, Proc. Amer. Math. Soc. **8** (1957), 535–536.
[5] N. H. MCCOY, *On quasi-commutative matrices*, Trans. Amer. Math. Soc. **36** (1934), no. 3, 327–340.
[6] F. V. SHIROKOV, *Proof of a conjecture by Kaplansky*, Uspehi Mat. Nauk, (N.S.) **11** (1956), 167–168 (in Russian).

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