NUMERICAL RANGES OF SOME FOGUEL OPERATORS

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Abstract. The Foguel operator is defined as $F_T = \begin{bmatrix} S^* & T \\ 0 & S \end{bmatrix}$, where S is the right shift on a Hilbert space $\mathcal{H}$ and T is an arbitrary bounded linear operator acting on $\mathcal{H}$. Obviously, the numerical range $W(F_0)$ of F_T with $T = 0$ is the open unit disk, and it was suggested by Gau, Wang and Wu in their Linear Algebra and Applications (2021) paper that $W(F_{aI})$ for non-zero $a \in \mathbb{C}$ might be an elliptical disk. In this paper, we described $W(F_{aI})$ explicitly and, as it happens, it is not.

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