INTERPOLATING SEQUENCES FOR THE BANACH ALGEBRAS GENERATED BY A CLASS OF TEST FUNCTIONS

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Abstract. The problem of characterizing interpolating sequences in a bounded domain $\Omega \subset \mathbb{C}^n$ for the Banach algebra $H^\infty(\Omega)$ of bounded holomorphic functions is well-studied in the literature. For the unit disc $\mathbb{D}$, the bidisc $\mathbb{D}^2$ and the symmetrized bidisc $\mathbb{G}^2$, there is a way to such a characterization via *the realization formula* that the function algebras $H^\infty(\Omega)$ possess in these cases. Our aim in this article is to present such a characterization of interpolating sequences in a more general setting for a class of Banach algebras that possess such a realization formula. The closed unit ball of these Banach algebras are known as the Schur–Agler-class associated to a class of test functions Ψ on Ω. We shall also note that the case of $\mathbb{D}$, $\mathbb{D}^2$ and $\mathbb{G}^2$ are special cases of our main result. A few other examples of function algebras is also mentioned where our main result applies leading to a characterization of interpolating sequences.

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