INEQUALITIES FOR THE WEIGHTED A–NUMERICAL RADIUS OF SEMI–HILBERTIAN SPACE OPERATORS

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Abstract. In this paper, we introduce the weighted A-numerical radius $\omega_{(A,\nu)}(\cdot)$ for semi-Hilbertian space operators. Further we obtain some basic properties and inequalities for $\omega_{(A,\nu)}(\cdot)$, which will be matched with earlier results about $\omega_A(\cdot)$. Moreover, we provide a refinement and generalization for inequalities obtained in [6, 16].

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