COWEN SETS FOR TOEPLITZ OPERATORS

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Abstract. The Cowen set for $\varphi \in L^\infty$ is defined by

$$\mathscr{E}(\varphi) = \left\{ k \in H^\infty : \|k\|_\infty \leqslant 1, \, \varphi - k\overline{\varphi} \in H^\infty \right\}.$$

It is known that the Toeplitz operator T_φ is hyponormal if and only if $\mathscr{E}(\varphi)$ is nonempty. In this paper, we study various properties of Cowen sets. Especially, we investigate in detail the case when $\mathscr{E}(\varphi)$ contains a constant and find a condition where $\mathscr{E}(\varphi)$ consists of exactly one constant.

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