Operators
and
Matrices
Volume 17, Number 2 (2023), 435–468
doi:10.7153/oam-2023-17-30

ON COPRODUCTS OF OPERATOR $\mathcal{A}$–SYSTEMS

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Abstract. Given a unital C^*-algebra $\mathcal{A}$, we prove the existence of the coproduct of two faithful operator $\mathcal{A}$-systems. We show that we can either consider it as a subsystem of an amalgamated free product of C^*-algebras, or as a quotient by an operator system kernel. We introduce a universal C^*-algebra for operator $\mathcal{A}$-systems and prove that in the case of the coproduct of two operator $\mathcal{A}$-systems, it is isomorphic to the amalgamated over $\mathcal{A}$, free product of their respective universal C^*-algebras. Also, under the assumptions of hyperrigidity for operator systems, we can identify the C^*-envelope of the coproduct with the amalgamated free product of the C^*-envelopes. We consider graph operator systems as examples of operator $\mathcal{A}$-systems and prove that there exist graph operator systems whose coproduct is not a graph operator system, it is however a dual operator $\mathcal{A}$-system. More generally, the coproduct of dual operator $\mathcal{A}$-systems is always a dual operator $\mathcal{A}$-system. We show that the coproducts behave well with respect to inductive limits of operator systems.

Mathematics subject classification (2020): 47L25, 46L07, 46L09.
Keywords and phrases: Operator system, coproduct, operator $\mathcal{A}$-system, C^*-cover, inductive limit.

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