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FURTHER NEW REFINEMENTS AND REVERSES OF REAL POWER FORM FOR YOUNG–TYPE INEQUALITIES VIA FAMOUS CONSTANTS AND APPLICATIONS

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Abstract. In this paper, we propose new refinements and reverses of real power form for Young-type inequalities, which generalizes the recent inspired results by D. Q. Huy et al. [Linear Algebra Appl. **656** (2023), 368-384], and by Y. Ren et al. [J. Inequal. Appl. **2020** (2020), Paper No. 98, 13 p.]. Furthermore, the above refinements and reverses are continued to improve via the famous constants consisting of Kantorovich constant and Specht ratio. As applications, we establish operator versions, inequalities for unitarily invariant norms and inequalities for determinants of matrices.

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