Operators and Matrices
Volume 17, Number 2 (2023), 517–524
doi:10.7153/oam-2023-17-33

A NON–INJECTIVE VERSION OF WIGNER'S THEOREM

MARK PANKOV AND LUCIJAN PLEVNIK

Abstract. Let H be a complex Hilbert space and let $\mathcal{F}_s(H)$ be the real vector space of all self-adjoint finite rank operators on H. We prove the following non-injective version of Wigner's theorem: every linear operator on $\mathcal{F}_s(H)$ sending rank one projections to rank one projections (without any additional assumption) is either induced by a linear or conjugate-linear isometry or constant on the set of rank one projections.

Mathematics subject classification (2020): 47B49, 15A86, 81P16.
Keywords and phrases: Projection, self-adjoint operator of finite rank, Wigner's theorem.

REFERENCES

[1] P. ANIELLO, D. CHRUŚCIŃSKI, *Symmetry witnesses*, J. Phys. A: Math. Theor. **50** (2017), no. 28, 16 p.
[2] V. BARGMANN, *Note on Wigner's theorem on symmetry operations*, J. Math. Phys. **5** (1964), 862–868.
[3] J. BARVÍNEK, J. HAMHALTER, *Linear algebraic proof of Wigner theorem and its consequences*, Math. Slovaca **67** (2017), 371–386.
[4] G. P. GEHÉR, *An elementary proof for the non-bijective version of Wigner's theorem*, Phys. Lett. A **378** (2014), 2054–2057.
[5] G. P. GEHÉR, *Wigner's theorem on Grassmann spaces*, J. Funct. Anal. **273** (2017), 2994–3001.
[6] G. P. GEHÉR, P. ŠEMRL, *Isometries of Grassmann spaces*, J. Funct. Anal. **270** (2016), 1585–1601.
[7] M. GYÖRY, *Transformations on the set of all n-dimensional subspaces of a Hilbert space preserving orthogonality*, Publ. Math. Debrecen **65** (2004), 233–242.
[8] J. S. LOMONT AND P. MENDELSON, *The Wigner unitary-antiunitary theorem*, Ann. Math. **78** (1963), 548–559.
[9] L. MOLNÁR, *Transformations on the set of all n-dimensional subspaces of a Hilbert space preserving principal angles*, Comm. Math. Phys. **217** (2001), 409–421.
[10] L. MOLNÁR, *Selected Preserver Problems on Algebraic Structures of Linear Operators and on Function Spaces*, Lecture Notes in Mathematics **1895**, Springer, 2007.
[11] L. MOLNÁR, *Maps on the n-dimensional subspaces of a Hilbert space preserving principal angles*, Proc. Amer. Math. Soc. **136** (2008), 3205–3209.
[12] M. PANKOV, *Orthogonal apartments in Hilbert Grassmanians*, Linear Algebra and its Applications **506** (2016), 168–182.
[13] M. PANKOV, *Geometric version of Wigner's theorem for Hilbert Grassmannians*, J. Math. Anal. Appl. **459** (2018), 135–144.
[14] M. PANKOV, *Wigner's type theorem in terms of linear operators which send projections of a fixed rank to projections of other fixed rank*, J. Math. Anal. Appl. **474** (2019), 1238–1249.
[15] M. PANKOV, *Wigner Type Theorems for Hilbert Grassmannians*, London Mathematical Society Lecture Note Series, vol. 460, Cambridge University Press, 2020.
[16] G. SARBICKI, D. CHRUŚCIŃSKI, M. MOZRZYMAS, *Generalising Wigner's theorem*, J. Phys. A: Math. Theor. **49** (2016), no. 30, 7 p.
[17] P. ŠEMRL, *Orthogonality preserving transformations on the set of n-dimensional subspaces of a Hilbert space*, Illinois J. Math. **48** (2004), 567–573.
[18] P. ŠEMRL, *Symmetries on bounded observables: a unified approach based on adjacency preserving maps*, Integral Equations Operator Theory **72** (2012), 7–66.

Paper OaM-17-33

[19] E. STØRMER, *Positive maps which map the set of rank k projections onto itself*, Positivity **21** (2017), 509–511.

[20] U. UHLHORN, *Representation of symmetry transformations in quantum mechanics*, Ark. Fys. **23** (1963), 307–340.

[21] V. S. VARADARAJAN, *Geometry of Quantum Theory* (2nd edition), Springer, 2000.

[22] E. P. WIGNER, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektrum*, Fredrik Vieweg und Sohn **1931** (english translation *Group Theory and its Applications to the Quantum Mechanics of Atomic Spectra*, Academic Press 1959).

Operators and Matrices
www.ele-math.com
oam@ele-math.com