

COMPUTING THE S -NUMERICAL RANGES OF DIFFERENTIAL OPERATORS

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Abstract. Following a number of recent studies of S -numerical range, we study the problem of computing the S -numerical range numerically for differential operators and block differential operators, particularly these of Sturm-Liouville type, Hain-Lüst type and Stokes type.

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