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FEEDBACK STABILIZATION OF THE LINEARIZED VISCOUS SAINT–VENANT SYSTEM BY CONSTRAINED DIRICHLET BOUNDARY CONTROL

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Abstract. In this paper, we study the stabilization of a linearized viscous Saint-Venant system by constrained Dirichlet boundary control in infinite time horizon. We proved the well posedness of the considered stabilization problem. Also, using an augmented state method, we were able to determine the optimal control (the constrained control) as a feedback control law. Moreover, thanks to the feedback control law, we proved the exponential stability of the solution to the linearized viscous Saint-Venant system, (defined by an unbounded operator). Some numerical experiments are given to illustrate the efficiency of the constrained Dirichlet boundary control.

Mathematics subject classification (2020): 93C20, 93D15, 93B52, 65L10.
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