CONVERSE OF FUGLEDE THEOREM

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Abstract. In this paper, we investigate when subnormal operators T_1 and T_2 are quasinormal provided their product is quasinormal. Also, we obtain as a corollary that subnormal n-th roots of a quasinormal operator are quasinormal, and thus we answer the question asked by Curto et al. in [4]. Also, we give sufficient conditions for quasinormal (subnormal) operators T_1 and T_2 to be normal if their product is normal. In other words, we find sufficient conditions for the converse of the Fuglede Theorem and also make a connection with the theory of subnormal pairs.

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