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SOME REFINEMENTS OF REAL POWER FORM INEQUALITIES FOR (p,h)–CONVEX FUNCTIONS VIA WEAK SUB–MAJORIZATION

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Abstract. The main goal of this paper, is to develop a general method for improving some new real power inequalities for (p,h)-convex and (p,h)-log-convex functions, which extends and unifies two recent and important results due to M. A. Ighachane and M. Bouchangour, (Filomat, **37** (16), (2023), 5259–5271) and (Operators and Matrices, **17** (1), (2023), 213–233). As applications of our results, we present further inequalities for the symmetric norms for τ-measurable operators.

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