

ON JOINT NUMERICAL RADIUS OF OPERATORS AND JOINT NUMERICAL INDEX OF A BANACH SPACE

ARPITA MAL

Abstract. Generalizing the notion of numerical range and numerical radius of an operator on a Banach space, we introduce the notion of joint numerical range and joint numerical radius of tuple of operators on a Banach space. We prove that on a finite-dimensional Banach space, the joint numerical radius can be retrieved from the extreme points. Furthermore, we introduce a notion of joint numerical index of a Banach space. We explore the same for direct sum of Banach spaces. Applying these results, finally we compute the joint numerical index of some classical Banach spaces.

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