A SYNCHRONOUS NPA HIERARCHY WITH APPLICATIONS

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Abstract. We present an adaptation of the NPA hierarchy to the setting of synchronous correlation matrices. Our adaptation improves upon the original NPA hierarchy by using smaller certificates and fewer constraints, although it can only be applied to certify synchronous correlations. We recover characterizations for the sets of synchronous quantum commuting and synchronous quantum correlations. For applications, we show that the existence of symmetric informationally complete positive operator-valued measures and maximal sets of mutually unbiased bases can be verified or invalidated with only two certificates of our adapted NPA hierarchy.

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