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SCALING POSITIVE DEFINITE MATRICES TO ACHIEVE PRESCRIBED EIGENPAIRS

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Abstract. We investigate the problem of scaling a given positive definite matrix A to achieve a prescribed eigenpair (λ, v), by way of a diagonal scaling D^*AD. We consider the case where D is required to be positive, as well as the case where D is allowed to be complex. We generalize a few classical results, and then provide a partial answer to a question of Pereira and Boneng regarding the number of complex scalings of a given 3×3 positive definite matrix A.

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