ORDER BOUNDEDNESS AND ESSENTIAL NORM OF GENERALIZED WEIGHTED COMPOSITION OPERATORS ON BERGMAN SPACES WITH DOUBLING WEIGHTS

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Abstract. In this paper, the order boundedness and essential norm of generalized weighted composition operators on Bergman spaces with doubling weights are characterized. Specially, we estimate the essential norm of these operators on weighted Bergman spaces by using the reduce order method.

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