

COMMUTING MAPS ON STRICTLY UPPER TRIANGULAR MATRIX RINGS

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Abstract. Let R be either a ring with 1 or a semiprime ring not necessarily with 1 and let $N_n(R)$ be the $n \times n$ strictly upper triangular matrix ring over R , where $n \geq 3$ is an integer. We completely characterize additive maps $f : N_n(R) \rightarrow N_n(R)$ satisfying $[f(x), x] = 0$ for all $x \in N_n(R)$. Our theorem naturally generalizes a recent result obtained by Bounds [3] for strictly upper triangular matrix rings over a field of characteristic 0.

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