SOME PROPERTIES OF EXACT PHASE RETRIEVABLE SUBSPACES

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Abstract. Sometimes a frame is not exact phase-retrievable, but it can be used to perform exact phase retrieval for some subsets of the Hilbert space. Hence the main purpose of this paper is to discuss some properties of the exact phase-retrievable subspaces. Firstly, according to the operator theory, the necessary and sufficient condition for a subspace to become an exact phase-retrievable subspace can be obtained. With the help of this property, the necessary and sufficient condition for the existence of the k-dimensional exact phase-retrievable subspace is discussed. Furthermore, we find that the subspace G of an exact phase-retrievable subspace M may not be an exact phase-retrievable subspace, although there is a frame that has the exact PR-redundancy property of G. Finally, the properties about the union of two exact phase-retrievable frames and the union of two exact PR subspaces are obtained.

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