Operators and Matrices
Volume 17, Number 4 (2023), 1065–1076
doi:10.7153/oam-2023-17-70

THE ESSENTIAL SPECTRUM EQUALITIES OF 2×2 UNBOUNDED UPPER TRIANGULAR OPERATOR MATRICES

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Abstract. Based on the space decomposition theory, the conditions for the essential spectrum equalities

$$\sigma_*(\mathscr{T}) = \sigma_*(A)\cup\sigma_*(D),\ (\sigma_* = \sigma_{\{e1,\ e2,\ e3,\ e4,\ e5,\ e6\}}),$$

for the diagonally dominant unbounded upper triangular block operator matrix $\mathscr{T} = \begin{pmatrix} A & B \\ 0 & D \end{pmatrix}$ are given, where the sets $\sigma_{e1}(\cdot)$ and $\sigma_{e2}(\cdot)$ denote the Gustafson and Weidmann essential spectrums, $\sigma_{e3}(\cdot)$ denotes Wolf essential spectrum, $\sigma_{e4}(\cdot)$ denotes the Schechter essential spectrum, $\sigma_{e5}(\cdot)$ and $\sigma_{e6}(\cdot)$ denote the essential approximation point spectrum and the essential defect spectrum, respectively.

Mathematics subject classification (2020): 47A53, 47A55.
Keywords and phrases: Upper triangular operator matrices, essential spectrums, *Fredholm* operators, spectral equalities.

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Paper OaM-17-70

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