CBMO ESTIMATES FOR SOME MULTILINEAR OPERATORS ON MIXED HERZ SPACES

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Abstract. In this paper, we develop the CBMO estimates for the multilinear operators of singular integral operator, fractional integral operator, and Hardy-type operators in higher dimensional cases, which include their commutators as special cases, on mixed Herz spaces. Furthermore, some endpoint cases are also obtained, such as Hardy-type and weak-type estimates for multilinear operators. Particularly, it is demonstrated that in some extreme cases, these operators are actually not bounded.

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