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HYPONORMALITY OF SPECIFIC UNBOUNDED PRODUCT OF DENSELY DEFINED COMPOSITION OPERATORS IN L^2 SPACES

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Abstract. Let $(X,\mathscr{A},\mu)$ be a σ-finite measure space. A transformation $\phi : X \to X$ is non-singular if $\mu \circ \phi^{-1}$ is absolutely continuous with respect with μ. For this non-singular transformation, the composition operator $C_\phi : \mathscr{D}(C_\phi) \to L^2(\mu)$ is defined by $C_\phi f = f \circ \phi$, $f \in \mathscr{D}(C_\phi)$.

For a fixed positive integer $n \geqslant 2$, basic properties of product $C_{\phi_n} \cdots C_{\phi_1}$ in $L^2(\mu)$ are conveyed in Section 3–5, including the dense definiteness, kernel, adjoint of (not necessarily bounded) $C_{\phi_n} \cdots C_{\phi_1}$. Under the assistance of these properties, when $C_{\phi_1}, C_{\phi_2}, \cdots, C_{\phi_n}$ are densely defined, hyponormality of specific (not necessarily bounded) $C_{\phi_n} \cdots C_{\phi_1}$ in $L^2(\mu)$ is characterized in Section 6.

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