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ON DISTANCE LAPLACIAN MATRICES OF WEIGHTED TREES

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Abstract. Let T be a weighted tree on n vertices and $D(T) := [[d_{ij}]]$ be the distance matrix of T. The distance Laplacian matrix of T is defined as

$$L_D(T) := \mathrm{Diag}(\sum_{j=1}^{n} d_{1j}, \ldots, \sum_{j=1}^{n} d_{nj}) - D(T).$$

We aim to show that all off-diagonal entries in the Moore-Penrose inverse of $L_D(T)$ are non-positive. Specifically, this result implies that the Moore-Penrose inverse of $L_D(T)$ is an $\mathbf{M}$-matrix.

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