

ON THE BEREZIN NUMBER OF OPERATOR MATRICES

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Abstract. Scalar quantities associated with Hilbert-space operators have attracted the attention of numerous researchers due to their role in understanding the geometry of the C^* -algebra of bounded linear operators on a Hilbert space. In this paper, we explore the Berezin number of operator matrices, and present several new relations that simulate the existing relations between the numerical radius and the operator norm.

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