

GENERALIZED CORE INVERSE IN BANACH $*$ -ALGEBRAS

HUANYIN CHEN AND MARJAN SHEIBANI*

Abstract. We introduce a new generalized inverse, called generalized core inverse in a Banach $*$ -algebra. This new inverse is an extension of weak core inverse defined for complex square matrix and bounded linear operators over Hilbert spaces. We present various characterizations of this new inverse. The relationship between the generalized core inverse and other generalized inverses is investigated. Finally, we consider the necessary and sufficient conditions under which generalized core inverse and generalized core-EP inverse coincide with each other in a Banach $*$ -algebra.

Mathematics subject classification (2020): 15A09, 16U99, 46H05.

Keywords and phrases: Core-EP inverse, generalized group inverse, weak group inverse, weak core inverse, Moore-Penrose inverse, Banach algebra.

REFERENCES

- [1] H. CHEN AND M. SHEIBANI ABDOLYUSEFI, *Theory of Clean Rings and Matrices*, Singapore: World Scientific, 2023.
- [2] H. CHEN AND M. SHEIBANI, *Generalized weighted core inverse in Banach algebras*, preprint, 2023, <https://doi.org/10.21203/rs.3.rs-3318243/v1>.
- [3] H. CHEN AND M. SHEIBANI, *On generalized core-EP invertibility in a Banach algebra*, arXiv:2309.09862 [math.FA], <https://doi.org/10.48550/arXiv.2309.09862>.
- [4] H. CHEN AND M. SHEIBANI, *Generalized group inverse in a Banach $*$ -algebra*, preprint, 2023, <https://doi.org/10.21203/rs.3.rs-3338906/v1>.
- [5] H. CHEN AND M. SHEIBANI, *Some new results in generalized weighted core inverse in Banach $*$ -algebras*, preprint, 2023, <https://doi.org/10.21203/rs.3.rs-3329741/v1>.
- [6] M. P. DRAZIN, *A class of outer generalized inverses*, *Linear Algebra Appl.*, **436** (2012), 1909–1923.
- [7] M. P. DRAZIN, *Commuting properties of generalized inverses*, *Linear Multilinear Algebra*, **61** (2013), 1675–1681.
- [8] Y. GAO AND J. CHEN, *Pseudo core inverses in rings with involution*, *Comm. Algebra*, **46** (2018), 38–50.
- [9] D. E. FERREYRA, F. E. LEVIS, A. N. PRIORI AND N. THOME, *The weak core inverse*, *Aequat. Math.*, **95** (2021), 351–373.
- [10] D. E. FERREYRA, V. ORQUERA AND N. THOME, *Representations of weighted WG inverse and a rank equation's solution*, *Linear and Multilinear Algebra*, **71** (2023), 226–241.
- [11] Z. FU, K. ZUO AND Y. CHEN, *Further characterizations of the weak core inverse of matrices and the weak core matrix*, *AIMS Math.*, **7** (2021), 3630–3647.
- [12] R. KUANG, C. DENG, *Common properties among various generalized inverses and constrained binary relations*, *Linear Multilinear Algebra*, **71** (2023), 1295–1322.
- [13] Y. LIAO, J. CHEN AND J. CUI, *Cline's formula for the generalized Drazin inverse*, *Bull. Malays. Math. Sci. Soc.*, **37** (2014), 37–42.
- [14] N. MIHAJLOVIC, *Group inverse and core inverse in Banach and C^* -algebras*, *Comm. Algebra*, **48** (2020), 1803–1818.
- [15] D. MOSIĆ, *Core-EP inverses in Banach algebras*, *Linear Multilinear Algebra*, **69** (2021), 2976–2989.
- [16] D. MOSIĆ AND J. MAROVT, *Weighted weak core inverse of operators*, *Linear Multilinear Algebra*, **70** (2022), 4991–5013.

- [17] D. MOSIĆ, P. S. STANIMIROVIC, *Expressions and properties of weak core inverse*, Appl. Math. Comput., **415** (2022), Article ID 126704, 23 p.
- [18] H. WANG, *Core-EP decomposition and its applications*, Linear Algebra Appl., **508** (2016), 289–300.
- [19] H. WANG, J. CHEN, *Weak group inverse*, Open Math., **16** (2018), 1218–1232.
- [20] M. ZHOU, J. CHEN AND Y. ZHOU, *Weak group inverses in proper *-rings*, J. Algebra Appl., **19** (2020), doi:10.1142/S0219498820502382.
- [21] M. ZHOU, J. CHEN, Y. ZHOU AND N. THOME, *Weak group inverses and partial isometries in proper *-rings*, Linear Multilinear Algebra, **70** (2021), 1–16.
- [22] Y. ZHOU AND J. CHEN, *Weak core inverses and pseudo core inverses in a ring with involution*, Linear Multilinear Algebra, **22** (2022), 6876–6890.
- [23] H. ZHU AND P. PATRICIO, *Characterizations for pseudo core inverses in a ring with involution*, Linear Multilinear Algebra, **67** (2019), 1109–1120.