RELATIVE RESIDUAL BOUNDS FOR EIGENVALUES
IN GAPS OF THE ESSENTIAL SPECTRUM

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Abstract. The relative distance between eigenvalues of the compression of a not necessarily semibounded self-adjoint operator to a closed subspace and some of the eigenvalues of the original operator in a gap of the essential spectrum is considered. It is shown that this distance depends on the maximal angles between pairs of associated subspaces. This generalises results by Drmač in [Linear Algebra Appl. **244** (1996), 155–163] from matrices to not necessarily (semi)bounded operators.

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