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ON THE PROPERTIES OF THE SYSTEMS OF ROOT VECTOR FUNCTIONS OF DIRAC–TYPE OPERATOR WITH SUMMABLE POTENTIAL

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Abstract. One-dimensional Dirac-type operator

$$Dy = By' + P(x)y, \quad y = (y_1, y_2)^T,$$

is considered in this work, where $B = \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix}$, $b_2 < 0 < b_1$, $P(x) = diag\,(p_1(x), p_2(x))$ and $p_j(x)$, $j = 1,2$ are the complex-valued functions defined on the arbitrary finite interval $G = (a,b)$ of the real axis with $p_j(x) \in L_1(G)$, $j = 1,2$.

We establish antiapriori estimates for associated vector functions. We also prove criterion of Bessel property and unconditional basis property for the systems of root vector functions of the operator D in $L_2^2(G)$.

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