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ON THE A_α SPECTRAL RADIUS AND A_α ENERGY OF NON–STRONGLY CONNECTED DIGRAPHS

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Abstract. Let $A_\alpha(G)$ be the A_α-matrix of a digraph G and $\lambda_{\alpha 1},\lambda_{\alpha 2},\ldots,\lambda_{\alpha n}$ be the eigenvalues of $A_\alpha(G)$. Let $\rho_\alpha(G)$ be the A_α spectral radius of G and $E_\alpha(G)=\sum_{i=1}^{n}\lambda_{\alpha i}^2$ be the A_α energy of G by using second spectral moment. Let $\mathcal{G}_n^m$ be the set of non-strongly connected digraphs with n vertices containing a unique strong component with m vertices and some directed trees hanging on each vertex of the strong component. In this paper, we characterize the digraph which has the maximal A_α spectral radius and the maximal (or minimal) A_α energy in $\mathcal{G}_n^m$.

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